Indian J. Phys. **74B (4)**, 267–270 (2000)

I J P B
— an international journal

Variation of selection effects with frequency in flux density—limited samples

G Anene and A C Ugwoke
Department of Physics and Industrial Physics, Nnamdi Azikiwe University, P.M B 5025 Awka, Anambra State, Nigeria
E-mail agabriel @ ictp.trieste it
anacc @ infoweb.abs.net

Received 6 January 1999, accepted 19 May 2000

Abstract : In this paper, we have analysed the variation of selection effects with frequency in flux density limited samples. Firstly, we estimated the contribution of selection effects in the observed variation of spectral index (α) with redshift (z) at radio luminosities estimated at 150 MHz, 2.5 GHz and 40 GHz respectively. Secondly, this analysis is repeated for the bolometric luminosity with the spectral index estimated at similar frequency ranges. The analyses were based on the 3CR high luminosity sources. The results show that in both cases, the contribution of selection effects in the observed $\alpha - (1 + z)$ variation, may be frequency dependent.

Keywords : Selection effects, flux density, luminosity

PACS Nos. : 11.30 Pb, 98 62 Qz, 98 62.Py

1. Introduction

The inconclusive results from the numerous correlation studies of spectral index (α) with radio luminosity (P) and/or redshift (z) have usually been attributed to the observed strong correlation of luminosity with redshift or due to radio K-correction [1–4]. It is known that in bright source samples the observed strong correlation between radio luminosity and redshift is usually attributed to selection effects. This arises because the source counts are so steep that most sources readily bunch up within a factor of 2–3 in flux density from the survey limit. The second factor which is related to the radio K-correction arises from estimating spectral indices at the observing, rather than the rest frequencies of the sources. This effect, which is frequency-dependent, becomes increasingly significant at high redshifts [5].

In the present paper, the authors have carried out a quantitative estimation of the dependence of spectral index on redshift resulting from the observed strong correlation between luminosity and redshift in flux density limited samples at 150 MHz, 2.5 GHz and 40 GHz (for radio luminosities). This procedure is repeated at the bolometric luminosity. The theoretical derivations of the relevant parameters used in the analysis, based on the 3CR steep spectra ($\alpha > 0.5$) source samples, are outlined in Section 2 while the results obtained and their analysis, are given in Section 3 [3.1 (radio) and 3.2 (bolometric)]. Finally, a summary of our results and conclusion are discussed in Section 4.

2. Variation of spectral index with redshift and luminosity in flux density—limited samples

In principle, the variation of spectral index with redshift independent of luminosity is best studied using samples of radio sources which in addition to having approximately the same median luminosities also have a wide range of redshifts. However, because of the observed strong correlation between luminosity and redshift, this has so far not been realised [6].

In this paper, it is shown that some quantitative estimates can be provided for the influence of selection effects in the observed spectral index-redshift ($\alpha - z$) relationship. This aspect of the correlation of α with z has so far remained largely uninvestigated in the literature.

In the present analysis, we shall assume a bivariate dependence of α on P and z of the sort [3]

$$\alpha(z, P_\nu) = \alpha_0 + a\log(1+z) + b\log(P_\nu/P^*), \quad (1)$$

$$\log P^* = 24,$$

where P is in units of WHz-1, where log P^* is normalizing luminosity P_ν, of a radio source of flux density, S_ν, at the observing frequency, ν depends on the two cosmological parameters—the Hubble's constant at the present epoch, H_0, and the density parameter, Ω according to the relation [7]

$$P_\nu = S_\nu\left(4c/H_0\Omega^2\right)^2(1+z)^{2Q(\Omega, z)+\alpha-1}, \quad (2)$$

where $(\alpha+1)$ is the term which expresses the spectral relativistic Doppler effect correction, c is the speed of light and

$$Q(\Omega, z) = \log 1/2(\Omega z + (\Omega - z))$$
$$\times\left((\Omega z + 1)^{1/2} - 1\right)/\log(1+z) \quad (3)$$

is a factor which expresses the inverse square law variation of luminosity with distance. Eq. (2) can be re-expressed in its most general form [8]

$$P_\nu = P_0(1+z)^\beta, \quad (4)$$

where $P_0 = S\left(4c/H_0\Omega^2\right)^2$ (5)

and $\beta = 2Q(\Omega, z) + \alpha - 1.$ (6)

In flux density limited samples, P_0 is determined by the flux density limit, S_ν, which equals the flux density of the sample and α is assumed to arise solely from selection effects. From the monovariation of α with P usually expressed as

$$\alpha(P_\nu) = \alpha_0 + b\log\left(P_\nu/P^*\right) \quad (7)$$

and substituting for P_ν, from eq. (4) into eq. (7), we get

$$\alpha(P_\nu) = \alpha_0 + \beta b\log(1+z) + b\log\left(P_0/P^*\right). \quad (8)$$

Similarly we may substitute eq. (4) in eq. (1) to obtain

$$\alpha(z, P_\nu) = \alpha_0 + a_\nu\log(1+z) + b\log\left(P_0/P^*\right), \quad (9)$$

where $a_\nu = (a+\beta b)$ is the magnitude of the observed $\alpha-(1+z)$ relationship. We can now carry out a quantitative analysis of the dependence of α on z and P since the product βb appearing in eq. (8) is the magnitude of the contribution of selection effects, as a result of the $P-(1+z)$ correlation, to the observed $\alpha-(1+z)$ data.

3. Results and analysis

3.1. Radio :

The present analyses were carried out using the well-defined 3CR samples of radio sources as compiled by [9], and published in [10].

The scope, however, shall be restricted only to the steep spectrum ($\alpha > 0.5$) and powerful $\log P_\nu > 24$ WHz-1 sources in the sample due to the difference in energy production mechanism between the flat ($\alpha < 0.5$) and steep ($\alpha > 0.5$) spectrum sources. We have adapted H_0 = 75 Km/s/Mpc, $\Omega = 1.0$, log P^* = 24 where P^* is in units of WHz-1 and α defined as positive *i.e.* $S_\nu \sim \nu^{-\alpha}$.

The sources were binned after making the assumption that in large data samples, the median~mean. To investigate the relative dependence of the spectral index on intrinsic luminosity and redshift, we fitted eq. (1) to the median value data in various ranges of luminosities and redshifts and obtained

$$\alpha(z, P_{0.15}) = 0.76 + 0.06\log(1+z)$$
$$+ 0.007\log(P_0 / P^*), \quad (10)$$

$$\alpha(z, P_{2.5}) = 0.62 + 0.51\log(1+z)$$
$$+ 0.06\log(P_0 / P^*), \quad (11)$$

$$\alpha(z, P_{40}) = 0.81 + 0.32\log(1+z)$$
$$+ 0.03\log(P_0 / P^*). \quad (12)$$

However, our main aim being to determine the magnitude of the contribution of the $P-(1+z)$ dependence on the observed $\alpha-(1+z)$ data, we have fitted a linear regression equation [see eq. (4)], of the form

$$\log P_\nu = \log P_0 + \beta\log(1+z) \quad (13)$$

for all those sources in our sample (*i.e.* for sources with $\alpha > 0.5$). The results are shown in Table 1.

Table 1. Results of regression analysis at ν = 0.15, 2.5 and 40 GHz.

ν (GHz)	β	log P_0 WHz-1	b
0.15	6.77	25.12	0.007
2.5	7.71	18.86	0.061
40	5.52	14.13	0.034

Substituting for β, P_0 and b from Table 1, with $\log P^*$ = 24 where P^* is in units of WHz-1, into eq. (8) for the 3 respective frequencies, we have

$$\alpha(P)_{0.15} = 0.76 + 0.047\log(1+z), \quad (14)$$

$$\alpha(P)_{2.5} = 0.62 + 0.47\log(1+z), \quad (15)$$

$$\alpha(P)_{40} = 0.81 + 0.19\log(1+z). \quad (16)$$

By comparing eqs. (14)–(16) with eqs. (10)–(12) according to their respective frequencies, we can see that the residual dependence of α on z, after subtracting that contributed by selection effects, are

$$\alpha(z)_{0.15} = 0.76 + 0.013\log(1+z), \quad (17)$$

$$\alpha(z)_{2.5} = 0.62 + 0.04\log(1+z), \quad (18)$$

$$\alpha(z)_{40} = 0.81 + 0.13\log(1+z). \quad (19)$$

Eqs. (17)–(19) are the expressions for the expected spectral steeping arising from redshift effects (and the radio K-correction) independent of all selection effects for all those sources with $\alpha > 0.5$ analysed in our sample at the three respective frequencies 150 MHz, 2.5 GHz and 40 GHz. It can be seen from eqs. (10)–(12) that the dependence of the spectral index on redshift is more than its dependence on luminosity, especially at high redshifts, ($z > 0.5$). Also by comparing eqs. (10)–(12) with eqs. (14)–(16) it follows that the observed $\alpha - z$ relationship at the three frequencies considered are respectively 78.3%, 92.2% and 59.4% showing that the variation of selection effects does not depend entirely on frequency since there is a drop at higher frequency (40 GHz).

3.2. At bolometric luminosity :

In this second part we shall investigate the effects of the variation of spectral index with frequency on the contributions of selection effects in the $\alpha-(1+z)$ correlation. It is shown that, here again, the $P-(1+z)$ relationship is also frequency dependent. The bolometric luminosity was measured for spectral indices estimated at three frequencies : 150 MHz, 2.5 GHz and 40 GHz in the 3 CR sample analysed. Since $\alpha = \alpha(\nu)$ at the bolometric luminosity, and adopting again a bivariate dependence of α on z and P of the sort [3],

$$\alpha(z, P) = \alpha_0 + a\log(1+z) + b\log(P_0/P^*) \tag{20}$$

and from [8], we have

$$P = P_{\rm bol}(1+z)^{\beta}. \tag{21}$$

If the observed variation of spectral index with redshift is assumed to arise from selection effects alone, then from the expression for the monovariation of α with P,

$$\alpha(P, \nu) = \alpha_0 + b\log(P_0/P^*) \tag{22}$$

and substituting for P in equation (22) from eq. (21), gives

$$\alpha(P, \nu) = \alpha_0 + \beta b\log(1+z) + b\log(P_{\rm bol}/P^*) \tag{23}$$

Also substituting similarly in eq. (20), it will give

$$\alpha(z, P) = \alpha_0 + a_\nu\log(1+z) + b\log(P_{\rm bol}/P^*), \tag{24}$$

where $\alpha_\nu = (a + \beta b)$ and represents the magnitude of the observed $\alpha-(1+z)$ data while βb is the contribution of selection effects in the same data.

Following the procedure adopted in § 3.1, we have estimated the contributions of selection effects to the observed $\alpha-(1+z)$ data for spectral index measured at 150 MHz, 2.5 GHz and 40 GHz (for the bolometric luminosity data) and obtained the following results :

$$\alpha_{0.15}(z, P) = 0.59 + 0.81\log(1+z) + 0.027\log(P_{\rm bol}/P^*) \tag{25}$$

$$\alpha_{0.15}(z, P) = 0.83 + 0.09\log(1+z) + 0.006\log(P_{\rm bol}/P^*) \tag{26}$$

$$\alpha_{40}(z, P) = 0.65 + 0.18\log(1+z) + 0.095\log(P_{\rm bol}/P^*) \tag{27}$$

To determine the magnitude of the contribution of selection effects (arising from the $P-(1+z)$ dependence) on the $\alpha-(1+z)$ data, we have fitted the linear regression equation, see eq. (13) for those sources in our sample with $\alpha > 0.5$ and obtained the following results tabled below :

Table 2. Results of regression analysis for α estimated at $\nu = 0.15$, 2.5 and 40 GHz.

ν (GHz)	β	(P_{bol}/P^*) WHz - 1	b
0.15	4.97	20.97	0.027
2.5	5.33	22.08	0.006
40	5.05	21.26	0.095

when these values in Table 2, together with $\log P^* = 34$ where P^* is in units of WHz - 1, are substituted in eq. (23) we obtained for the respective frequencies,

$$\alpha_{0.15}(P) = 0.59 + 0.134\log(1+z) \tag{28}$$

$$\alpha_{2.5}(P) = 0.83 + 0.032\log(1+z) \tag{29}$$

$$\alpha_{40}(P) = 0.65 + 0.480\log(1+z) \tag{30}$$

By comparing eqs. (28)–(30) with eqs. (25)–(27) it is deducible that the dependence of spectral index on redshift, after subtracting that contributed by selection effects are respectively,

$$\alpha_{0.15}(z) = 0.59 + 0.676\log(1+z), \tag{31}$$

$$\alpha_{2.5}(z) = 0.83 + 0.060\log(1+z), \tag{32}$$

$$\alpha_{0.15}(z) = 0.65 - 0.300\log(1+z). \tag{33}$$

Eqs. (31)–(33) are the expressions for redshif effects independent of selection effects in those sources with $\alpha > 0.5$ analysed in our sample.

4. Discussion and conclusions

The results obtained from both Subsections 3.1 and 3.2 of our analysis, show that the dependence of spectral index on redshift is more than its dependence on luminosity, especially at higher redshifts ($z > 0.5$). This is clear from eqs. (10)–(12) and (25)–(27). Again, in Subsection 3.1, by comparing eqs. (10)–(12) with eqs. (14)–(16), we can see that the contributions of selection effects to the observed ($\alpha - z$) relationship (at the radio spectrum) are ~78.3, 92.2 and 59.4% at the frequencies of 50 MHz, 2.5 GHz and 40 GHz, respectively. These results seem to suggest no clear trend in the $\alpha-(1+z)$ variation with frequency. However, at the bolometric luminosity, considered in part 2 of our analysis, the trend shows an increase of this effect with frequency of ~16.54, 35.56 and 26.76% respectively, at the three frequencies considered.

Several authors have previously worked extensively on the correlation of spectral index with redshift. For example, Laing and Peacock [1] found a correlation between α and P for the bright source samples they analysed, though, however, it was not then possible to decide whether the primary correlation was with luminosity or redshift due to the strong selection effects present in their sample. Allington-Smith [2] showed that at low flux levels α correlates more with luminosity than with redshift provided that sources with very steep frequency spectra were excluded. Gopal [5] using two samples of powerful sources with approximately the same median luminosities but with different median redshifts, again showed that α depends more on luminosity than on redshift. Another similar result was obtained by Onuora [6] using a much larger source sample. These last two contributions were, however, based on a rather restricted redshift plane.

A somewhat different result was obtained by Windhorst *et al* [3] based also on a large number of samples, though at low flux levels. They were able to separate completely the effects of luminosity from those of redshift (in the observed $\alpha - z$ correlation) and showed that α correlates more strongly with redshift than with radio luminosity. It may be noted however, that in the absence of any strong cosmological luminosity evolution, their result could be attributed to radio K-correction. Again, Kapahi and Kulkarni [4] used two point spectral indices, between rest frequencies of 1.4 and 2.7 GHz in radio galaxies from 1.4 and 2.7 GHz surveys to show that only weak and marginal $\alpha - P$ and $\alpha - z$ correlations could be found. They put a very strong argument to show that the much stronger correlation of spectral index with luminosity (or redshift) observed in bright source samples, should arise jointly from the contributions of radio K-correction and selection effects. The contributions of the above effects arise from a steeper slope of the luminosity function at higher luminosities. This is due to very steep-spectrum, high redshift sources that are usually preferentially included in low frequency surveys.

In the present analysis, we have shown that at the radio spectrum, there is indeed a strong contribution from selection effects to the observed $\alpha - z$ correlation of ~78.3, 92.2 and 59.4% at 150 MHz, 2.5 GHz and 40 GHz surveys, respectively in the 3 CR sources. Also this contribution is ~16.54, 35.56 and 26.76% at the bolometric luminosity. These two results show independently that selection effects contribute very significantly to the observed $\alpha - z$ correlation, irrespective of the possible contributions from the radio K-correction and may have a dependency on frequency.

Acknowledgments

The first author would like to thank the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. Financial support from the Swedish International Development Cooperation Agency is also acknowledge.

References

[1] R A Laing and J A Peacock *Mon Not. Roy Astron. Soc* **190** 903 (1980)

[2] J R Allington-Smith *Mon Not. Roy Astron Soc* **209** 665 (1984)

[3] R A Windhorst, D Mathis and L Newschaefer *The Edwin Hubble Centinnial Symp ASP Conf Ser* (Brigham Young, Utah) Vol 10 (ed) R Kron *et al* p 389 (1989)

[4] V K Kapahi and V K Kulkarni *Astron J* **97** 1 (1990)

[5] K Gopal *Astron Astrophys* **192** 37 (1988)

[6] L I Onuora *Astrophys Space Sci* **162** 349 (1989)

[7] L I Onuora and S E Okoye *Astrophys J* **270** 370 (1983)

[8] A A Ubachukwu, S E Okoye and L I Onuora *Nigerian Acad Sci* **5** 52 (1993)

[9] R A Laing, J M Riley and M S Longair *Mon Not. Roy Astron Soc* **204** 15 (1983)

[10] T Herbig and A C S Readhead *Astrophys J. Supply Ser* **81** 83 (1992)